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# EXTERNAL HEAT TRANSFER—CALCULATION METHODS AND MOIST MATERIAL DRYING KINETICS

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**Abstract**—The method, based on the use of the Rebinder number, for calculating the duration of drying and the heat transfer is set forth and the relationships for the Nusselt numbers in the first and second periods are given.

## NOMENCLATURE

$T_a$, ambient temperature;
$T_w$, wet-bulb temperature;
$T_r$, radiation temperature;
$r$, latent heat of evaporation;
$\rho_0$, density;
$R_v$, hydraulic radius;
$Rb$, Rebinder number;
$u_e$, equilibrium moisture content.

As is known [1–3], the heat-transfer coefficient $\alpha_m$ may be constant only within the period of a constant rate of drying, while in the falling period it diminishes with a decreasing moisture content.

To calculate $\alpha_m$, Lebedev [2] suggested the relationship

$$\frac{\alpha_m}{\alpha_{cr}} = \left(\frac{W}{W_{cr}}\right)^n, \tag{1}$$

and to obtain the heat-transfer Nusselt number, the relationship

$$Nu = ARe^p\left(\frac{T_a}{T_w}\right)^m\left(\frac{T_r}{T_a}\right)^k\left(\frac{W}{W_{cr}}\right)^n. \tag{2}$$

We have attempted to evaluate the Nusselt number ($Nu$) for the second period on the basis of its value in the first period ($Nu_{cr}$) and the dimensionless rate of drying $N^*$. These values can be readily found both theoretically and experimentally. In order to obtain the heat-transfer coefficient by relation (2), the knowledge of the surface material temperature is required. However, its exact definition as well as of the power exponents for every material is often difficult.

The basic equation of the drying kinetics is of the form

$$q_f(\tau) = \rho_0 R_v r\frac{d\bar{u}}{d\tau}(1+Rb). \tag{3}$$

The rate of drying in the first period can be determined, provided the heat flux is known, by the following formula

$$N = \frac{q_f}{r\rho_0 R_v}. \tag{4}$$

Then in the first period of drying the heat-transfer intensity is governed by the Newton formula

$$N = \frac{q_f}{r\rho_0 R_v} = \frac{\alpha_{cr}(T_a - T_w)}{r\rho_0 R_v}. \tag{5}$$

In the second, by the formula

$$\frac{d\bar{u}}{d\tau} = -HN(\bar{u}-u_e). \tag{6}$$

The relative drying coefficient depends on the material properties and initial moisture content and is defined as

$$H = \frac{1}{u_{crr} - u}. \tag{7}$$

In the period of the falling rate of drying, the heat flux is specified by the Newton formula for convective heat transfer

$$q_f = \bar{\alpha}(T_a - T_s). \tag{8}$$

Substituting (6) and (7) into equation (3), yields

$$\bar{\alpha}(T_a - T_s) = \rho_0 R_v r H N(\bar{u}-u_e)(1+Rb). \tag{9}$$

Replacement of $N$ by expression (5) gives

$$\bar{\alpha}(T_a - T_s) = H(\bar{u}-u_e)\alpha_{cr}(T_a - T_w)(1+Rb),$$

or

$$\frac{\bar{\alpha}(T_a - T_s)}{\alpha_{cr}(T_a - T_w)} = H(\bar{u}-u_e)(1+Rb). \tag{10}$$

Denoting

$$\Delta T^* = \frac{T_a - T_s}{T_a - T_w}; \quad N^* = \frac{du/d\tau}{N}, \tag{11}$$

and substituting the results into relation (10), we obtain

$$\Delta T^* = N^*(1+RC)\frac{\alpha_{cr}}{\alpha}, \tag{12}$$

or

$$\Delta T^* = N^*(1+Rb)\frac{Nu_{cr}}{Nu}. \tag{13}$$

For such materials as caprone, the Rebinder number ranges from 0.0034 to 0.015 at $u$ = 0.02–0.08 kg/kg; for ceramic plate ($\delta = 6.5$ mm), from 0.03 to 0.3 at $u = 0.1$–$0.03$ kg/kg and for asbestos, from 0.02 to 0.3 at $u = 0.02$–$0.18$ kg/kg. In case the Rebinder number is small,

$$\Delta T^* = N^* \frac{Nu_{cr}}{Nu}, \qquad (14)$$

but

$$\frac{Nu_{cr}}{Nu} = f(N^*).$$

As seen from Fig. 1, all the experimental points, irrespective of the drying regimes, fall on one curve

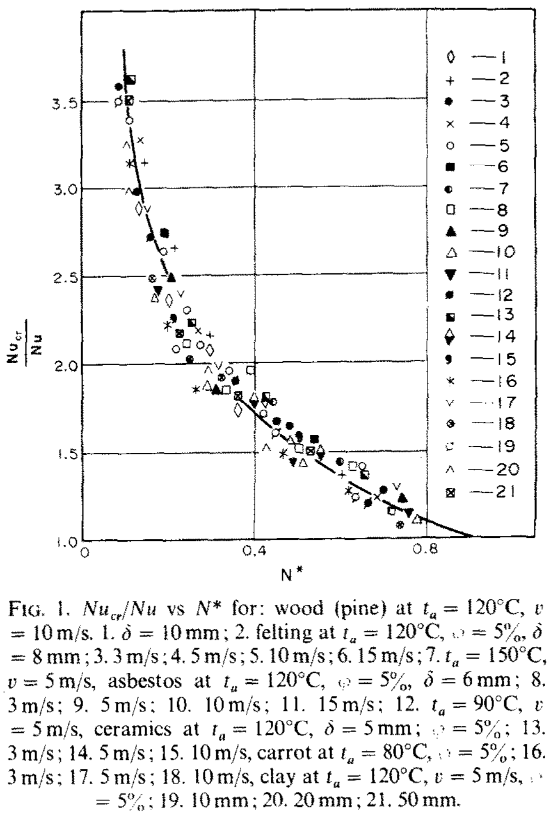


FIG. 1. $Nu_{cr}/Nu$ vs $N^*$ for: wood (pine) at $t_a = 120°C$, $v = 10$ m/s. 1. $\delta = 10$ mm; 2. felting at $t_a = 120°C$, $\varphi = 5\%$, $\delta = 8$ mm; 3. 3 m/s; 4. 5 m/s; 5. 10 m/s; 6. 15 m/s; 7. $t_a = 150°C$, $v = 5$ m/s, asbestos at $t_a = 120°C$, $\varphi = 5\%$, $\delta = 6$ mm; 8. 3 m/s; 9. 5 m/s; 10. 10 m/s; 11. 15 m/s; 12. $t_a = 90°C$, $v = 5$ m/s, ceramics at $t_a = 120°C$, $\delta = 5$ mm; $\varphi = 5\%$; 13. 3 m/s; 14. 5 m/s; 15. 10 m/s, carrot at $t_a = 80°C$, $\varphi = 5\%$; 16. 3 m/s; 17. 5 m/s; 18. 10 m/s, clay at $t_a = 120°C$, $v = 5$ m/s, $\varphi = 5\%$; 19. 10 mm; 20. 20 mm; 21. 50 mm.

described by equation (15) (the uncertainty being 10–15%)

$$\frac{Nu_{cr}}{Nu} = N^{*-0.57}, \qquad (15)$$

or

$$Nu = Nu_{cr} N^{*-0.57}. \qquad (16)$$

The reported investigations [4] on moist plate-fluidized bed heat transfer have shown that the data are well approximated by equation (16).

Thus, formula (16) is a general one for many colloid capillary-porous materials permitting estimation of the Nusselt number in the second period at $Bi_q \leq 1$.

With allowance for the Rebinder number formula (15) becomes

$$(1+Rb)\frac{Nu_{cr}}{Nu} = N^{*-0.57}. \qquad (17)$$

Substitution of formula (17) into expression (13) yields

$$\Delta T^* = N^{*0.43}, \qquad (18)$$

and

$$T_s = T_a - N^{*0.43}(T_a - T_w). \qquad (19)$$

The drying rate being known, equation (19) allows estimation of the material surface temperature in the second period, which is not always reasonable to obtain experimentally.

The laws governing internal and external heat transfer prove to be in such a complex interrelation that derivation of analytical expressions for the kinetics of drying of some particular material involves great difficulties.

Therefore, of considerable interest for design calculations of drying are approximate, sufficiently reliable relations with the minimum quantity of constants to be determined experimentally. Most advanced are the approaches based on the general laws governing the kinetics of drying and correlation of a great amount of experimental data. Among them are the methods of Luikov [1] and Krasnikov [5].

Let us consider some laws governing the process of drying and approximate methods used for predicting the heat-transfer kinetics resulting from the analysis of experimental data on drying of a number of capillary-porous and colloid materials with different modes of heat supply.

The treatment of experimental data [5] has shown that the heat-transfer intensity in the period of the falling rate of drying changes with respect to time with a sufficient accuracy according to the exponential relation

$$q_{II} = q_I \exp(-m_1 \tau_{II}), \qquad (20)$$

where $\tau_{II}$ is the drying time in the second period related to zero.

Denoting the ratio of the heat fluxes in the periods of the falling and constant rates of drying by $q^*$ and the ratio of the times of drying by $\tau^*$, transforms expression (20) to a dimensional form

$$q^*_{(\tau)} = \frac{q_{II}}{q_I} = \exp\left(-m\frac{\tau_{II}}{\tau_I}\right) = \exp(-m\tau^*), \qquad (21)$$

where $m_1$ and $m$ are constants determined experimentally.

On the other hand, the governing equation of drying kinetics, which relates the heat-transfer $q^*$ and moisture transfer $N^*$, is of the form

$$q^*_{(\tau)} = \frac{q_{II}}{q_I} = N^*(1+Rb). \qquad (22)$$

From expressions (21) and (22), we have

$$N^*(1+Rb) = \exp(-m\tau^*). \qquad (23)$$

The Rebinder number for many of the capillary-porous materials in a considerable range of moisture contents varies from 0.01 to 0.3, its maximum values (0.2–0.3) corresponding to the end of drying (i.e. to

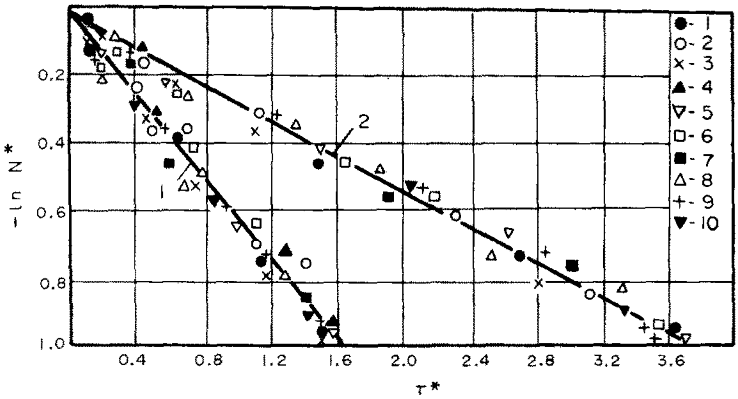


FIG. 2. $N^*$ vs $\tau^*$ at forced-convection drying of asbestos (1) and felting (2) plates. 1. $t_a = 90°C$; 2. $t_a = 120°C$; 3. $t_a = 150°C$ (at $v = 5$ m/s, $\varphi = 5\%$, $\delta = 8$ mm); 4. $v = 3$ m/s; 5. 10 m/s; 6. 15 m/s; 7. 20 m/s; 8. 25 m/s (at $t_a = 120°C$, $\varphi = 5\%$, $\delta = 8$ mm); 9. $\delta = 6$ mm; 10. $\delta = 12$ and 18 mm (at $t_a = 120°C$, $v = 3$ m/s, $\varphi = 5\%$).

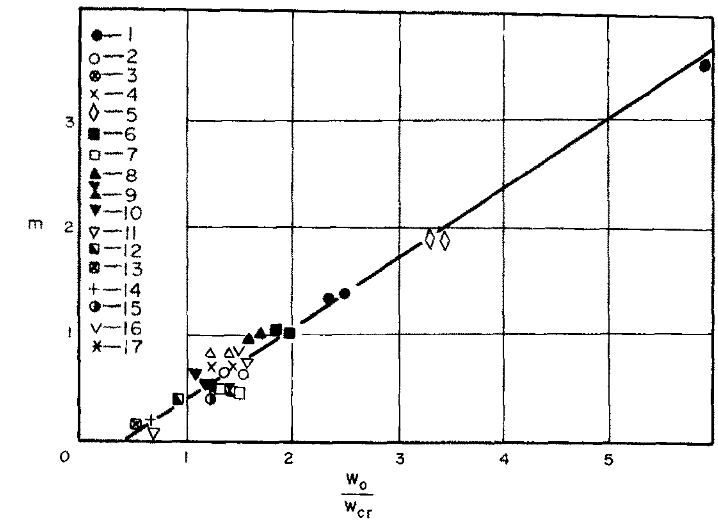


FIG. 3. Constant $m$ vs $W_0/W_{cr}$. 1. asbestos; 2. felting; 3. grass; 4. leather; 5. felt; 6. ceramics; 7. cardboard; 8. clay; 9. peat slab; 10. yeast; 11. carrot; 12. sunflower seeds; 13. potato; 14. beet; 15. cement stone; 16. macaroni; 17. microbiological mass of lysine.

moisture contents close to equilibrium). The neglect of small $Rb$ results in

$$N^* = \exp(-m\tau^*). \qquad (24)$$

Correlation of the experimental data on drying of different materials by $\ln N^* = f(\tau^*)$ (Fig. 2) has shown that the experimental points fall satisfactorily on the straight lines for the whole range of operating parameters ($t_a$, $\varphi$, $v$) and the material thicknesses. The constant $m$ in formula (24) is a linear function of the moisture content ratio ($W_0/W_{cr}$) for the whole class of the moist materials considered (Fig. 3).

$$m = 0.67\frac{W_0}{W_{cr}} - 0.35. \qquad (25)$$

Relationship (25) has been obtained for $2 \leqslant v \leqslant 5$, $90 \leqslant t \leqslant 150°C$. Thus, equation (24) has the constant $m$ whose meaning resembles the relative coefficient of drying $H$.

For the materials whose drying proceeds only in the period of the falling rate, the experimental data are correlated by

$$N^* = \exp(-m\tau). \qquad (26)$$

In this case the constant $m$ has the dimension l/time units.

The drying rate in the second period can be found from equation (24)

$$\frac{dW}{d\tau} = N\exp(-m\tau^*) \qquad (27)$$

which upon integration within the prescribed range gives

$$-(W_{cr} - W) = N\frac{\tau_I}{m}\left|\exp\left(-\frac{m}{\tau_I}\tau_{II}\right) - \right|. \qquad (28)$$

Upon transformations, we get the time of drying in the period of the falling rate

$$\tau_{II} = -\frac{2.3\tau_I}{m}\ln\left|1 - \frac{(W_{cr}-W)}{N\tau_I}\right|. \quad (29)$$

Taking into account the drying time in the first period

$$\tau_I = \frac{W_0 - W_{cr}}{N} \quad (30)$$

we determine the overall duration of the process by

$$\tau = \tau_I + \tau_{II} = \frac{W_0 - W_{cr}}{N} - \frac{2.3(W_0 - W_{cr})}{Nm} \times \ln\left|1 - \frac{(W_{cr}-W)m}{(W_0 - W_{cr})}\right|, \quad (31)$$

or, finally,

$$\tau = \frac{W_0 - W_{cr}}{N}\left\{1 - \frac{2.3}{m}\ln\left[1 - \frac{(W_{cr}-W)m}{(W_0 - W_{cr})}\right]\right\} \quad (32)$$

where $W$ is the instantaneous moisture content of the material.

For the materials whose drying does not involve the constant rate period, the overall drying time is determined by integration of equation (26)

$$\tau = -\frac{2.3}{m}\ln\left|1 - \frac{(W_0 - W)m}{N}\right|, \quad (33)$$

where

$$N = \left(\frac{dW}{d\tau}\right)_{max}$$

is the maximum rate of drying at the initial time.

The comparison of the results given by equations (23) and (24) has shown that, in fact, the constant $m$ is independent of the Rebinder number up to 0.2–0.3. Herein, only the scatter of experimental data decreases, that is the accuracy of calculations becomes higher.

With high rate of drying in "severe" conditions, when the value of the Rebinder number cannot be ignored, the overall time of the process is determined by

$$\tau = \frac{W_0 - W_{cr}}{N} \times \left\{1 - \frac{2.3}{m}\ln\left[1 - \frac{(W_{cr}-W)m}{(W_0 - W_{cr})}(1+Rb)\right]\right\}. \quad (34)$$

By substituting $N^*$ from (24) into formula (19), we arrive at the relation to determine the material surface temperature for the convective mode of drying by the known $m$

$$T_s = T_a - \exp(-0.43m\tau^*)(T_a - T_w). \quad (35)$$

The maximum uncertainty in the calculations of the drying time and the material temperature with neglect of the Rebinder number is 7 $\pm 10\%$ for mild regimes, which is quite admissible in practice.

The relations obtained are necessary to define the quantitative drying characteristics (time, heat-transfer rate, material temperature).

The above methods, just like that reported in [5], allow one curve-based calculations of the drying time and material temperature at any regime, provided the rate of drying in the first period is known.

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## METHODES DE CALCUL DU TRANSFERT DE CHALEUR EXTERNE ET DE LA CINETIQUE DE SECHAGE D'UN MATERIAU HUMIDE

**Résumé**—On développe la méthode, basée sur l'utilisation du nombre de Rebinder, du calcul de la durée de séchage et du transfert thermique. On donne les expressions du nombre de Nusselt dans les première et seconde périodes.

## BERECHNUNGSMETHODEN FÜR DEN ÄUßEREN WÄRMEÜBERGANG UND DIE KINETIK DER TROCKNUNG FEUCHTER MATERIALIEN

**Zusammenfassung**—Die Methode zur Berechnung der Dauer der Trocknung und des Wärmeübergangs, die auf der Verwendung der Rebinder-Zahl beruht, wird erweitert, und die Beziehungen für die Nusselt-Zahl in der ersten und zweiten Periode werden angegeben.

## МЕТОДЫ РАСЧЕТА ВНЕШНЕГО ТЕПЛООБМЕНА И КИНЕТИКА ПРОЦЕССА СУШКИ ВЛАЖНЫХ МАТЕРИАЛОВ

**Аннотация** — В статье излагается метод расчета продолжительности процесса сушки, а также теплообмена на основе введенного числа Ребиндера и приводятся экспериментально полученные зависимости о связи между критериями Нуссельта во втором и первом периодах.